

C.U.SHAH UNIVERSITY

Summer Examination-2017

Subject Name: Transform Theory

Subject Code: 4SC05TTE1

Branch: B.Sc.(Mathematics)

Semester: 5

Date: 01/04/2017

Time: 02:30 To 05:30

Marks: 70

Instructions:

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

- Q-1 Attempt the following questions: (14)**
- a) State Dirichlet's condition. (02)
 - b) If $f(x) = x \cos x$ in $(-\pi, \pi)$ then find value of a_n (01)
 - c) What is period of $\sin pt$? (01)
 - d) State first shifting theorem. (02)
 - e) Find Laplace transform of $e^{4t} + \cos at$. (02)
 - f) If $L(f(t)) = \bar{f}(s)$, then $L\left(\int_0^t f(u)du\right) = \underline{\hspace{2cm}}$ (01)
 - g) $L^{-1}\left[\frac{1}{(s-a)^n}\right] = \underline{\hspace{2cm}}$ (01)
 - h) Define Z – transform. (02)
 - i) Derive Z – transform of $\frac{1}{n!}$ (02)

Attempt any four questions from Q-2 to Q-8

- Q-2 Attempt all questions (14)**
- a) State and prove Euler's formulae for Fourier series expansion of a function $f(x)$. (07)
 - b) State and prove Convolution theorem. Apply convolution theorem to evaluate $L^{-1}\left\{\frac{1}{s(s-1)}\right\}$. (07)
- Q-3 Attempt all questions (14)**
- a) Obtain Fourier series for the function $f(x) = x$, in the interval $0 < x < 2\pi$. (07)
 - b) Find Laplace inverse transform of $\frac{1}{(s+1)(s-2)(s-3)}$. (07)
- Q-4 Attempt all questions (14)**
- a) Find Fourier cosine and sine integral of $f(x) = e^{-kx}$ ($x > 0, k > 0$) (07)



- b) (i) Find finite Fourier cosine transform of $f(x) = 2x, 0 < x < 4$. (07)
(ii) Find finite Fourier sine transform of $f(x) = lx - x^2, 0 \leq x \leq l$.

Q-5 Attempt all questions (14)

- a) Prove that (06)
(a) $Z(a^n) = \frac{z}{z-a}$
(b) $Z(n^p) = -z \frac{d}{dz} Z(n^{p-1}), p$ is a positive integer
b) State and prove Damping rule (05)
c) Find Z- transform of $2 \cos n\theta + 3 \sin n\theta + n^2$ (03)

Q-6 Attempt all questions (14)

- a) Find Laplace transform of the following (07)
(i) $e^{-3t} \sin^2 t$
(ii) $\frac{\sin t}{t}$
(iii) $t^2 \sin 4t$
b) Using Laplace transform, find the solution of the initial value problem (07)
 $y'' - 4y' + 4y = 64 \sin 2t, y(0) = 0, y'(0) = 1$

Q-7 Attempt all questions (14)

- a) Find Fourier series expansion of function $f(x) = x^2, -\pi \leq x \leq \pi$. Hence, find (07)
sum $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \dots$.
b) Find Fourier half range cosine series of the function (07)
 $f(t) = 2t, 0 < t < 1$
 $= 2(2-t), 1 < t < 2$

Q-8 Attempt all questions (14)

- a) Obtain Fourier series for the function $f(x) = \pi x \quad 0 \leq x \leq 1$ (06)
 $= \pi(2-x) \quad 1 \leq x \leq 2$
where $p = 2l = 2$.
b) Find Fourier sine transform of $f(x) = 0 \quad 0 < x < a$ (05)
 $= x \quad a \leq x \leq b$
 $= 0 \quad x > b$
c) Evaluate: $\int_0^\infty t e^{-3t} \cos t dt$ (03)

